

# Corrections

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Equation (25) writes

$$(\mathbf{H}_n^{kk})^{-1} = \frac{1}{\mathbf{e}_k^\top \boldsymbol{\Sigma}_n^{-1} \mathbf{e}_k} \left( \mathbf{I}_N - \frac{\mathbf{h}_n^{[k]} (\mathbf{h}_n^{[k]})^\top}{\mathbf{e}_k^\top \boldsymbol{\Sigma}_n^{-1} \mathbf{e}_k + ((\mathbf{h}_n^{[k]})^\top \mathbf{w}_n^{[k]})^{-2}} \right).$$

The correct formula should be

$$(\mathbf{H}_n^{kk})^{-1} = \frac{1}{\mathbf{e}_k^\top \boldsymbol{\Sigma}_n^{-1} \mathbf{e}_k} \left( \mathbf{I}_N - \frac{\mathbf{h}_n^{[k]} (\mathbf{h}_n^{[k]})^\top}{1 + (\mathbf{e}_k^\top \boldsymbol{\Sigma}_n^{-1} \mathbf{e}_k) ((\mathbf{h}_n^{[k]})^\top \mathbf{w}_n^{[k]})^2} \right). \quad (1)$$

*Proof.* Substituting the  $k$ th entry of the score function given by Eqn. (17) into Eqn. (12), we have

$$\begin{aligned} \mathbf{H}_n^{kk} &= \frac{\partial^2 \mathcal{J}_{IVA}}{\partial \mathbf{w}_n^{[k]} \partial (\mathbf{w}_n^{[k]})^\top} = \frac{1}{T} \mathbf{X}^{[k]} (\mathbf{X}^{[k]})^\top \mathbf{e}_k^\top \boldsymbol{\Sigma}_n^{-1} \mathbf{e}_k + \frac{\mathbf{h}_n^{[k]} (\mathbf{h}_n^{[k]})^\top}{((\mathbf{h}_n^{[k]})^\top \mathbf{w}_n^{[k]})^2} \\ &= \mathbf{e}_k^\top \boldsymbol{\Sigma}_n^{-1} \mathbf{e}_k \mathbf{I}_N + \frac{\mathbf{h}_n^{[k]} (\mathbf{h}_n^{[k]})^\top}{((\mathbf{h}_n^{[k]})^\top \mathbf{w}_n^{[k]})^2}. \end{aligned}$$

Note that the last equality stems from the fact that  $\mathbf{X}^{[k]}$  is whitened. Using matrix inversion lemma, i.e.,

$$(\mathbf{A} + \mathbf{UCV})^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1} \mathbf{U} (\mathbf{C}^{-1} + \mathbf{VA}^{-1} \mathbf{U})^{-1} \mathbf{VA}^{-1},$$

the rank-1 update of a scaled identity matrix is given by

$$(c\mathbf{I} + \mathbf{uu}^\top)^{-1} = \frac{1}{c} \left( \mathbf{I} - \frac{\mathbf{uu}^\top}{c + \mathbf{u}^\top \mathbf{u}} \right). \quad (2)$$

Using (2) with  $c = \mathbf{e}_k^\top \boldsymbol{\Sigma}_n^{-1} \mathbf{e}_k$  and  $\mathbf{u} = \frac{\mathbf{h}_n^{[k]}}{((\mathbf{h}_n^{[k]})^\top \mathbf{w}_n^{[k]})}$ , we obtain (1).  $\square$