Correction. Equation (25) writes

$$(\boldsymbol{H}_n^{kk})^{-1} = \frac{1}{\boldsymbol{e}_k^\top \boldsymbol{\Sigma}_n^{-1} \boldsymbol{e}_k} \left(\boldsymbol{I}_N - \frac{\boldsymbol{h}_n^{[k]} (\boldsymbol{h}_n^{[k]})^\top}{\boldsymbol{e}_k^\top \boldsymbol{\Sigma}_n^{-1} \boldsymbol{e}_k + \big((\boldsymbol{h}_n^{[k]})^\top \boldsymbol{w}_n^{[k]} \big)^{-2}} \right).$$

The correct formula should be

$$(\boldsymbol{H}_n^{kk})^{-1} = \frac{1}{\boldsymbol{e}_k^\top \boldsymbol{\Sigma}_n^{-1} \boldsymbol{e}_k} \left(\boldsymbol{I}_N - \frac{\boldsymbol{h}_n^{[k]} (\boldsymbol{h}_n^{[k]})^\top}{1 + (\boldsymbol{e}_k^\top \boldsymbol{\Sigma}_n^{-1} \boldsymbol{e}_k) \big((\boldsymbol{h}_n^{[k]})^\top \boldsymbol{w}_n^{[k]} \big)^{2}} \right). \tag{1}$$

Proof. Substituting the kth entry of the score function given by Eqn. (17) into Eqn. (12), we have

$$\begin{aligned} \boldsymbol{H}_n^{kk} = \frac{\partial^2 \mathcal{J}_{IVA}}{\partial \boldsymbol{w}_n^{[k]} \partial (\boldsymbol{w}_n^{[k]})^\top} &= \frac{1}{T} \boldsymbol{X}^{[k]} (\boldsymbol{X}^{[k]})^\top \boldsymbol{e}_k^\top \boldsymbol{\Sigma}_n^{-1} \boldsymbol{e}_k + \frac{\boldsymbol{h}_n^{[k]} (\boldsymbol{h}_n^{[k]})^\top}{\big((\boldsymbol{h}_n^{[k]})^\top \boldsymbol{w}_n^{[k]} \big)^2} \\ &= \boldsymbol{e}_k^\top \boldsymbol{\Sigma}_n^{-1} \boldsymbol{e}_k \boldsymbol{I}_N + \frac{\boldsymbol{h}_n^{[k]} (\boldsymbol{h}_n^{[k]})^\top}{\big((\boldsymbol{h}_n^{[k]})^\top \boldsymbol{w}_n^{[k]} \big)^2}. \end{aligned}$$

Note that the last equality stems from the fact that $\boldsymbol{X}^{[k]}$ is whitened. Using matrix inversion lemma, i.e.,

$$(\boldsymbol{A} + \boldsymbol{U}\boldsymbol{C}\boldsymbol{V})^{-1} = \boldsymbol{A}^{-1} - \boldsymbol{A}^{-1} \boldsymbol{U} \big(\boldsymbol{C}^{-1} + \boldsymbol{V} \boldsymbol{A}^{-1} \boldsymbol{U} \big)^{-1} \boldsymbol{V} \boldsymbol{A}^{-1},$$

the rank-1 update of a scaled identity matrix is given by

$$(c\boldsymbol{I} + \boldsymbol{u}\boldsymbol{u}^\top)^{-1} = \frac{1}{c} \left(\boldsymbol{I} - \frac{\boldsymbol{u}\boldsymbol{u}^\top}{c + \boldsymbol{u}^\top \boldsymbol{u}} \right). \tag{2}$$

Using (2) with $c = \boldsymbol{e}_k^\top \boldsymbol{\Sigma}_n^{-1} \boldsymbol{e}_k$ and $\boldsymbol{u} = \frac{\boldsymbol{h}_n^{[k]}}{(\boldsymbol{h}_n^{[k]})^\top \boldsymbol{w}_n^{[k]}}$, we obtain (1). □